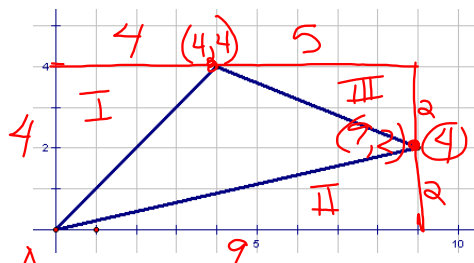


Area by Determinants

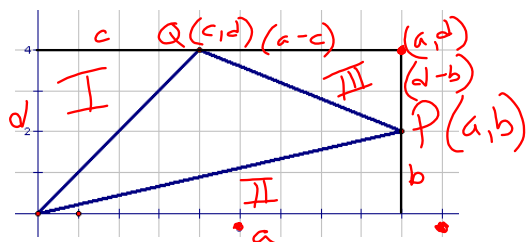


$$A_{\text{Rect}} - A_I - A_{II} - A_{III}$$

$$36 - 8 - 9 - 5$$

$$A = 14u^2$$

$$\frac{1}{2} \begin{vmatrix} 9 & 2 \\ 4 & 4 \end{vmatrix} = 14u^2$$



$$A_{\text{Rect}} - I - II - III$$

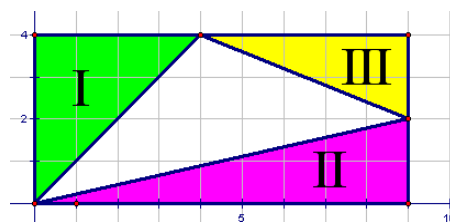
$$a \cdot d - \frac{1}{2}cd - \frac{1}{2}ab - \frac{1}{2}(a-c)(d-b)$$

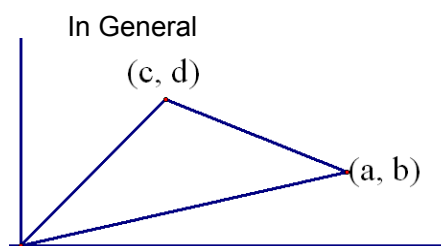
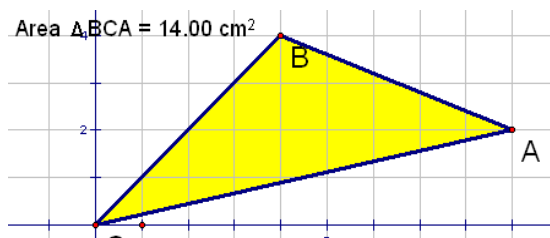
$$ad - \frac{1}{2}cd - \frac{1}{2}ab - \frac{1}{2}(ad - ab - cd + bc)$$

$$\frac{1}{2}ad - \frac{1}{2}bc$$

$$= \frac{1}{2}(ad - bc)$$

$$= \frac{1}{2} \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$





Example:

Find the area

$(0, 0), (5, 2), (4, 6)$

$A = \frac{1}{2} \begin{vmatrix} 5 & 2 \\ 4 & 6 \end{vmatrix} = \frac{1}{2} (30 - 8) = \frac{1}{2} (22) = 11$

$A = 11 \text{ u}^2$

Do:

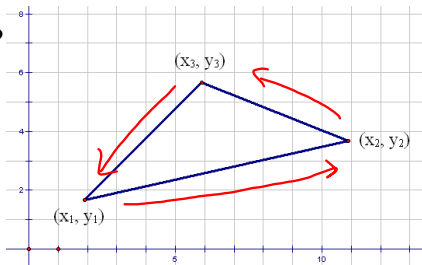
Find the area.

$(0, 0), (5, -3), (-10, 9)$

$A = \frac{1}{2} \begin{vmatrix} 5 & -3 \\ -10 & 9 \end{vmatrix} = \frac{1}{2} (45 - 30) = \frac{1}{2} (15) = 7.5$

$A = 7.5 \text{ u}^2$

What if...?



$$\frac{1}{2} \left[\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} + \begin{vmatrix} x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} \right]$$

Ex:

Find the area:

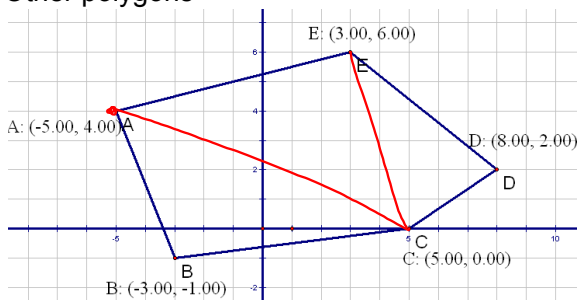
(3, 8) (5, 1) (-1, 2)

$$= \frac{1}{2} \left[\begin{vmatrix} 3 & 8 \\ 5 & 1 \end{vmatrix} + \begin{vmatrix} 5 & 1 \\ -1 & 2 \end{vmatrix} + \begin{vmatrix} -1 & 2 \\ 3 & 8 \end{vmatrix} \right]$$

$\begin{matrix} 40 & -1 & 6 \\ -37 & 10 & -14 \end{matrix}$

$A = 20_{u^2}$

Other polygons



$$A = \frac{1}{2} \left[\begin{vmatrix} -5 & 4 \\ -3 & -1 \end{vmatrix} + \begin{vmatrix} -3 & -1 \\ 5 & 0 \end{vmatrix} + \begin{vmatrix} 5 & 0 \\ 8 & 2 \end{vmatrix} + \begin{vmatrix} 8 & 2 \\ 3 & 6 \end{vmatrix} + \begin{vmatrix} 3 & 6 \\ -5 & 4 \end{vmatrix} \right]$$

$\begin{matrix} 17 & 5 & 10 & 42 & 42 \\ \frac{1}{2} 116 \\ A = 58_{u^2} \end{matrix}$